Chapter-1

REAL NUMBERS

Fundamental Theorem of Arithmetic:

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex:
$$30 = 2 \times 3 \times 5$$

LCM and HCF: If a and b are two positive integers. Then the product of a, b is equal to the product of their LCM and HCF.

LCM×HCF=
$$a \times b$$

To Find LCM and HCF of 12 and 18 by the prime factorization method.

$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

HCF of 12 and
$$18 = 2^1 \times 3^1 = 6$$

(Product of the smallest powers of each common prime factors in the numbers)

LCM of 12 and
$$18 = 2^2 \times 3^2 = 36$$

(Product of the greatest powers of each prime factors in the numbers)

Product of the numbers = $12 \times 18 = 216$

$$LCM \times HCF = 36 \times 6 = 216$$

- \therefore Product of the numbers = LCM×HCF
 - Natural numbers Set N= {1,2,3,4,-----}
 - Whole number Set $W = \{0,1,2,3,4,----\}$

Rational numbers (Q): If p, q are whole numbers and $q\neq 0$ then the numbers in the form \underline{p}

of q are called **Rational numbers**.

Rational numbers Set
$$Q = \{\frac{p}{q} | p, q \in z, q \neq 0 | HCF(p,q) = 1\}$$

All rational numbers can be written either in the form of terminating decimals or non-terminating repeating decimals.

Ex:
$$-\frac{2}{7}, \frac{5}{2}, -4, 3, 0, ----$$

Between two distinct rational numbers there exist infinite number of rational numbers.

A rational number between 'a' and 'b' = $\frac{a+b}{2}$

Terminating Decimals in Rational Numbers:

Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form, $\frac{p}{q}$ where p and q are co-prime, and the prime factorization of q is of the from 2^n . 5^m , where n,m are non-negative integers.

Conversely

Let x= be a $\frac{p}{q}$ rational number, such that the prime factorization of q is of the form $2^n.5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminate.

Ex: In the rational number $\sqrt[3]{20}$, p=3, q=20 $q=20=2\times2\times5=2^2\times5^1$ in the form of 2^n5^m .

 $\frac{3}{20}$ Is in the form of terminating decimal.

and
$$\frac{3}{20} = \frac{3}{2^2 \times 5^1} = \frac{3 \times 5}{2^2 \times 5^2} = \frac{15}{(10)^2} = \frac{15}{100} = 0.15$$

Non-terminating, Recurring Deceimals In Rational Numbers:

Let x = be a $\frac{p}{q}$ rational number, Such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion. which is non-terminating repeating (recurring).

Ex : In the rational number , p=11, q=30 $q = 30 = 2 \times 3 \times 5$ is not in the form of $2^n 5^m$

 $\frac{11}{30}$ is non-terminating, repeating decimal.

Irrational Numbers (Q¹):

The numbers cannot written in the form of $\frac{p}{q}$ are called irrational numbers.

The decimal expansion of every irrational numbers is non-terminating and repeating.

Ex:
$$\pi$$
, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{12}$, 0.1011011001100 -----

• An irrational number between a and $b = \sqrt{ab}$ \sqrt{p} is irrational, where P is Prime.

Ex:
$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11} - - - -$$

Let P be a prime number. Let p divides a^2 . Then P divides a, where a is a positive integer.

- Sum (or difference) of a rational number and an irrational number is an irrational numbers.
- Product (or quotient) of non-zero rational and an irrational number is an irrational numbers.
- The Sum of the two irrational numbers need not be irrational. $\sqrt{2}$, $-\sqrt{2}$ are irrational but $\sqrt{2} + (-\sqrt{2}) = 0$, Which is rational.
- Product of two irrational number need not be irrational.

Ex: $\sqrt{2}, \sqrt{8}$ are irrational but $\sqrt{2}.\sqrt{8} = \sqrt{16} = 4$ which is rational.

Real Numbers (R):

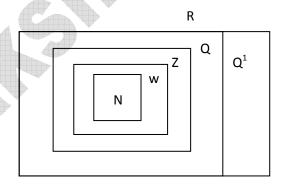
The Set of rational and irrational numbers together are called real numbers. $R = Q U Q^{1}$

- Between two distinct real numbers there exists infinite number of real number.
- Between two distinct real numbers there exists infinite number of rational and irrational number.
- With respect to addition Real numbers are Satisfies closure, Commutative, Associative, Identity, Inverse and Distributive properties.

Here '0' is the additive identity and additive inverse of a is - a

 With respect to multiplication, non-zero real numbers are Satisfies closure, Commutative, Associative, Identity, Inverse properties.
 Here '1' is the multiplicative identity.

For a
$$(\neq 0) \in \mathbb{R}$$
, $\frac{1}{a}$ is the multiplication inverse of 'a'.
$$N \subset W \subset Z \subset Q \subset \mathbb{R}$$



Logarithms:

Logarithms are used for all sort of calculations in engineering, Science, business and economics.

If $a^n = x$; we write it as $\log_a x = n$, where a and x are positive numbers and $a \ne 1$.

Logarithmic form of $a^n = x$ is $\log_a x = n$

Exponential form of $\log_a x = n$ is $a^n = x$

Ex: Logarithmic form of 4^3 =64 is $\log_4 64$ =3

Ex: Exponential form of $\log_4 64=3$ is $4^3=64$.

The logarithms of the same number to different bases are different

Ex: $\log_4 64 = 3$, $\log_8 64 = 2$

The logarithm of 1 to any base is zero i.e. $\log_a 1 = 0$, $\log_2 1 = 0$

The logarithm of any number to the same base is always one.

i.e.
$$\log_{a} a = 1, \log_{10} 10 = 1$$

Laws of logarithms:

(1).
$$\log_a x y = \log_a x + \log_a y$$

(2).
$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

(3).
$$\log_a x^m = m.\log_a x$$

The logarithm of a number consists of two parts.

- (i). The integral part of the logarithm (Characteristic).
- (ii). The fractional or decimal part of the logarithm (Mantissa)

Ex:
$$\log_{10}16 = 1.2040$$

Characteristic
$$= 1$$

Essay Type Questions:

(1). Prove that $\sqrt{3}$ is irrational.

Solution: Since we are using proof by contradiction

Let us assume the contrary

i.e
$$\sqrt{3}$$
 is rational.

If it is rational, then there exist two integers

a and b, b\neq 0, Such that
$$\sqrt{3} = \frac{a}{b}$$
(1)

and also a,b are co-Prime (i.e. HCF(a, b)=1)

$$\sqrt{3} = \frac{a}{b}$$

Squaring on both sides, we get

$$3b^2 = a^2$$
(2)

 \therefore 3 divide a^2

3 divides a (\therefore p is a Prime number. If p divided a^2 , then p divides a. where a is positive integers)

So, we can write

a = 3k for some integer k.

Substituting in equation (2), we get

$$3b^2 = (3k)^2$$

$$3b^2 = 9k^2$$

$$b^2 = 3k^2$$

 $3 \text{ divides } b^2$

... P is a prime number. If p divided a², then p divides a.)

: a and b have at have at least 3 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This means that our Supposition is wrong.

Hence, $\sqrt{3}$ is an irrational.

(2). Prove that 3+2 $\sqrt{5}$ is irrational (Reasoning Proof)

Solution: Let us assume, to the contrary, that 3+2 $\sqrt{5}$ is rational.

i.e, we can find co-primes a and b,

$$b\neq 0$$
 such that $3+2$ $\sqrt{5} = \frac{a}{b}$

$$\frac{a}{b} - 3 = 2\sqrt{5}$$

$$\frac{a-3b}{b} = 2\sqrt{5}$$

$$\frac{a-3b}{2b} = \sqrt{5}$$

Since a and b are integers, we get $\frac{a-3b}{2b}$ is rational

$$(\because \frac{a-3b}{2b} = \sqrt{5})$$

So $\sqrt{5}$ is rational

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has fact arisen because of our incorrect assumption that $3+2\sqrt{5}$ rational.

So we conclude that $3 + 2\sqrt{5}$ is irrational.

(3). Prove that $\sqrt{5} + \sqrt{2}$ is irrational (Reasoning Proof)

Solution: Since we are using proof by contradiction

Let us assume the contrary

i.e
$$\sqrt{5} + \sqrt{2}$$
 rational

Let
$$\sqrt{5} + \sqrt{2} = \frac{a}{b}$$
, where a, b are integers and $b \neq 0$.

$$\sqrt{2} = \frac{a}{b} - \sqrt{5}$$

Squaring on the both sides

$$(\sqrt{2})^2 = (\frac{a}{b} - \sqrt{5})^2$$

$$2 = \frac{a^2}{b^2} - 2\sqrt{5} \frac{a}{b} + 5$$

$$2\sqrt{5}\frac{a}{b} = \frac{a^2}{b^2} + 5 - 2$$

$$2\sqrt{5}\frac{a}{b} = \frac{a^2}{b^2} + 3$$

$$2\sqrt{5}\frac{a}{b} = \frac{a^2 + 3b^2}{b^2} \qquad \qquad \sqrt{5} = \frac{a^2 + 3b^2}{b^2} \cdot \frac{b}{2a}$$

$$\sqrt{5} = \frac{a^2 + 3b^2}{b^2} \cdot \frac{b}{2a}$$

$$\sqrt{5} = \frac{a^2 + 3b^2}{2ab}$$

Since a, b are integers $\frac{a^2 + 3b^2}{2ab}$ is rational and so, $\sqrt{5}$ is rational.

This contradict the fact that $\sqrt{5}$ is irrational.

Hence $\sqrt{5} + \sqrt{2}$ is irrational.

(4). Prove the first law of logarithms (Reasoning proof)

Solution:

The first law of logarithms states

$$\log_a x y = \log_a x + \log_a y$$

Let $x = a^n$ and $y = a^m$ where a > 0 and $a \ne 1$.

Then we know that we can write

Log_a
$$x = n$$
 and $loga $y = m$ (1)$

Using the first law of exponents we know that

$$a^n . a^m = a^{m+n}$$

$$x \cdot y = a^n \cdot a^m = \frac{a^{n+m}}{a}$$

i.e
$$x y = a^{n+m}$$

writing in the logarithmic form, we Know that

$$\log_a x y = n+m$$
>(2)

$$\log_a x y = \log_a x + \log_a y$$
 (from (1))

Model Question: Prove the Second law of logarithms

$$log_a = \frac{x}{y} = log_a = x - log_a y$$

(5). Prove the third law of logarithms states (Reasoning proof)

Solution:

The third law of logarithms states

$$\log_a x^{m} = m \log_a x$$

Let
$$x = a^n$$
 so $\log_a x = n$ (1)

Suppose ,we raise both sides of $x=a^n$ to the power m, we get

$$X^{m} = (a^{n})^{m}$$

$$x^m = a^{nm}$$
 (Using the laws of exponents)

$$x^{m} = a^{nm}$$

Writing in the logarithmic form, we get

$$Log_a x^m = nm$$

=m n

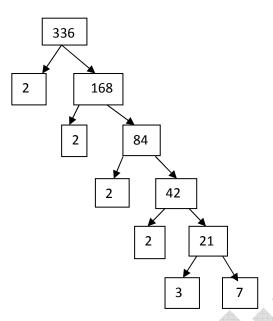
m. $\log_a x$ (from equation (1))

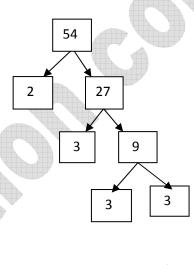
$$\log_a x^m = m \log_a x$$

Short Answer Question:

Q(1). Find the LCM and HCF of the number 336 and 54 and verify that LCM×HCF =product of the two numbers.

Solution:





$$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$$

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$$

There fore

LCM (336, 54) = Product of the greatest power of each prime factors, in the number $= 2^4 \times 3^3 \times 7 = 3024$

HCF (336, 54) = Product of the smallest power of each common prime factors, in the numbers

$$= 2 \times 3 = 6$$

Verification:

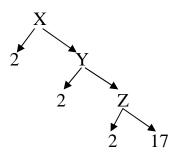
 $LCM \times HCF = Product of two numbers$

 $3024 \times 6 = 336 \times 54$

18144 = 18144

Hence Verified.

Q(2). Find the value of X, Y and Z in the following factor tree. Can the value of 'x' be found without finding the value of 'Y' and 'Z' If yes, Explain.



Solution: From the factor tree

$$z = 2 \times 17 = 34$$

$$y = 2 \times z = 2 \times 34 = 68$$

$$x = 2 \times y = 2 \times 68 = 136$$

Yes, the value of the x can be found without finding the value of 'y' and 'z' as follow:

$$x = 2 \times 2 \times 2 \times 17 = 136$$

Q(3). Sita takes 35 seconds to pack and label a box. For Ram, the same job takes 42 seconds and for Geeta, it takes 28 seconds. If they all start using labeling machines at the same time, after how many seconds will they be using the labeling machines together?

(communication)

Solution:

Required number of seconds is the LCM of 35, 42 and 28

$$35 = 5 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$28 = 2 \times 2 \times 7$$

LCM of 35, 42 and $28 = 2^2 \times 3 \times 5 \times 7$

Hence Sita, Ram and Geeta will be using the labeling machines together after 420 seconds, i.e 7 minutes.

Q(4). Explain why $(17\times11\times2) + (17\times11\times5)$ is a composite number?

(Reasoning proof)

Solution: $(17 \times 11 \times 2) + (17 \times 11 \times 5)$

$$17 \times 11 \times (2+5) = 17 \times 11 \times 7$$

Since $(17\times11\times2) + (17\times11\times5)$ can be expressed as product of prime, it is a composite number.

Q(5). Without actual division, State whether the following rational number are terminating or non-terminating, repeating decimal.

(i).
$$\frac{15}{200}$$

(ii).
$$\frac{64}{455}$$

Solution:

(I).
$$\frac{15}{200} = \frac{3 \times 5}{40 \times 5} = \frac{3}{40} = \frac{3}{2^3 \times 5}$$

Here $q = 2^3 \times 5^1$, which is of the form $2^n.5^m$

(n=3, m= 1). So, the rational number $\frac{15}{200}$ has a terminating decimal expansion.

(II) .
$$\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$$

Here $q = 5 \times 7 \times 13$, which is not the form $2^n.5^m$. So ,the rational number Has a non-terminating repeating decimal expansion.

(6). White the decimal expansion of the following rational number without actual division. (communication)

(i)
$$\frac{143}{110}$$
 (II). $\frac{16}{3125}$

(i).
$$\frac{143}{110} = \frac{11 \times 13}{11 \times 10} = \frac{13}{10} = 0.3$$

(ii).
$$\frac{16}{3125} = \frac{16}{5^5} = \frac{16 \times 2^5}{5^5 \times 2^5} = \frac{16 \times 32}{(5 \times 2)^5} = \frac{512}{10^5}$$

$$=\frac{512}{100000}=0.00512$$

(7). Prove that $\frac{2\sqrt{3}}{5}$ is an irrational (Reasoning proof)

Solution: Let us assume, to the contrary, that $\frac{2\sqrt{3}}{5}$ is rational. Then there exist Co-prime positive integers 'a' and 'b' such that

$$\frac{2\sqrt{3}}{5} = \frac{a}{b}$$

$$\sqrt{3} = \frac{5a}{2b}$$

5, a, 2, b, are integers, $\therefore \frac{5a}{2b}$ is a rational numbers)

This contradicts the fact that $\sqrt[7]{3}$ is irrational.

So, our assumption is not correct

Hence $\frac{2\sqrt{3}}{5}$ is an irrational number.

(8). Determine the value of the following(Problem solving)

(i)
$$\text{Log }_2 \frac{1}{16}$$

(2)
$$\operatorname{Log}_{x} \sqrt{3}$$

(i) Let
$$\log_2 \frac{1}{16} = t$$
, then the exponential form is $2^t = \frac{1}{16}$

$$2^{t} = \frac{1}{2^{4}}$$

$$2^{t} = 2^{-4}$$

$$t = -4$$

$$\log_2 \frac{1}{16} = -4$$

(ii) Let
$$\log_x \sqrt{x} = t$$
, then the exponential form is

$$x^t = \sqrt{x}$$

$$\mathbf{x}^{t} = \mathbf{x}^{\frac{1}{2}}$$

$$t = \frac{1}{2}$$

$$\log_{x} \sqrt{x} = \frac{1}{2}$$

- (9) Write each of the following expression as log N.
- (i) $2 \log 3 3 \log 2$ (ii) $\log 10 + 2 \log 3 \log 2$ (problem solving)

$$2 \log 3 - 3 \log 2 = \log 3^2 - \log 2^3$$
 (: $m \log x = \log x^m$)

$$= \log 9 - \log 8$$

$$= \log \frac{9}{8} \quad (\because \log x - \log y = \log \frac{x}{y})$$

(ii)
$$\log 10 + 2 \log 3 - \log 2 = \log 10 + \log 3^2 - \log 2$$
 (: $m \log x = \log x^m$) = $= \log 10 + \log 9 - \log 2$

$$\log(10\times9) - \log 2 \quad (\because \log x + \log y = \log xy)$$

$$=\log 90 - \log 2$$

$$=\log \frac{90}{}$$
 (: $\log x - \log y = \log \frac{x}{}$)

(10). Expand the following

(i)
$$\log \frac{128}{625}$$

(ii)
$$\log \sqrt{\frac{x^3}{y^2}}$$

(i)
$$\log \frac{128}{625} = \log 128 - \log 625$$

$$= \log 2^7 - \log 5^4$$

= $7 \log 2 - 4 \log 5$

 $(: \log x^{m} = m \log x)$

(ii)
$$\log \sqrt{\frac{x^3}{y^2}} = \log(\frac{x^3}{y^2})^{\frac{1}{2}}$$

$$= \log \frac{x^{\frac{3}{2}}}{y}$$

$$=\log x^{\frac{3}{2}} - \log y$$

$$\frac{3}{2}\log x - \log y$$

Very Short Answer Question:

(i) Find any rational number between the numbers

$$3\frac{1}{3}$$
 and $3\frac{2}{3}$ (problem sloving)

Slove:

Given numbers $3\frac{1}{3}$ and $3\frac{2}{3}$

$$= \frac{10}{3} \text{ and } \frac{11}{3}$$

The rational numbers between 'a' and 'b' is $\frac{a+b}{2}$

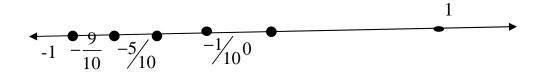
The rational numbers between $3\frac{1}{3}$ and $3\frac{2}{3}$ is $\frac{3\frac{1}{2}+3\frac{2}{3}}{2}$

$$=\frac{7}{2}$$

$$= 3.5$$

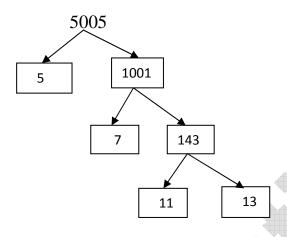
(2) Represent the number $\frac{9}{10}$ on the number line. (Visualization and Representation)

Solution:



(3) Express the numbers 5005 as a product of its prime factors. (communication)

Solve:



So,
$$5005 = 5 \times 7 \times 11 \times 13$$

(4). Given that HCF (306, 657) =9. Find LCM (306, 657) (problem solving)

Solve:

LCM × HCF =Product of numbers

$$LCM(306,657) = \frac{306 \times 657}{HCF(306,657)} = \frac{306 \times 657}{9} = 22338$$

(5). Check whether 6ⁿ can end with the digit 0 for any natural number 'n'.

(Reasoning poof)

Solution: we know that any positive integers ending with the digit zero is divisible by 5 and so its prime factorization must contain the prime 5.

We have

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

The only primes in the factorization of 6ⁿ are 2 and 3

There are no other primes in the factorization of 6^n

(By uniqueness of the fundamental Theorem of Arithmetic)

5 does not occur in the prime factorization of 6^n for any n.

6ⁿ does not end with the digit zero for any natural number n.

(6). Write (i) $3^5 = 243$ (ii) $10^{-3} = 0.001$ in the logarithmic form. (Communication)

- (i). The logarithmic form of $3^5 = 243$ is $\log_3 243 = 5$
- (ii) The logarithmic form of $10^{-3} = 0.001$ is $\log_{10} 0.001 = -3$

(7). Write (i) $\log_4 64 = 3$

(ii) $\log_a \sqrt{x} = b$ in exponential form (Communication)

Solve:

- (i). The exponential form of $\log_4 64 = 3$ is $4^3 = 64$
- (ii). The exponential form of $\log_a \sqrt{x} = b$ is $a^b = \sqrt{x}$

(8). Expand log15 (Problem solving)

Solution: $\log 15 = \log (3 \times 5) = \log 3 + \log 5$ ($\because \log x \ y = \log x + \log y$)

(9). Explain why 3×5×7+7 is a composite number. (Reasoning proof)

Solution:
$$3 \times 5 \times 7 + 7 = 7(3 \times 5 \times 1 + 1) = 7 \times (16) = 2 \times 2 \times 2 \times 2 \times 7$$
$$= 2^{4} \times 7$$

Since $3\times5\times7+7$ can be expressed as a product of primes therefore by fundamental theorem of Arithmetic it is a composite no.

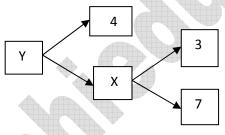
Multiple Choice Questions

(1). The prime factor of $2\times7\times11\times17\times23+23$ is

(A) 7

- (B) 11
- (C) 17
- (D) 23
- (2). A physical education teacher wishes to distribute 60 balls and 135 bats equally among a number of boys. Find the greatest number receiving the gift in this way......
 - (A) 6

- (B) 12
- (C) 18
- (D) 15
- (3). The Values of x and y in the given figure are []



- (A) X = 10, Y = 14
- (B) X=21, Y=84

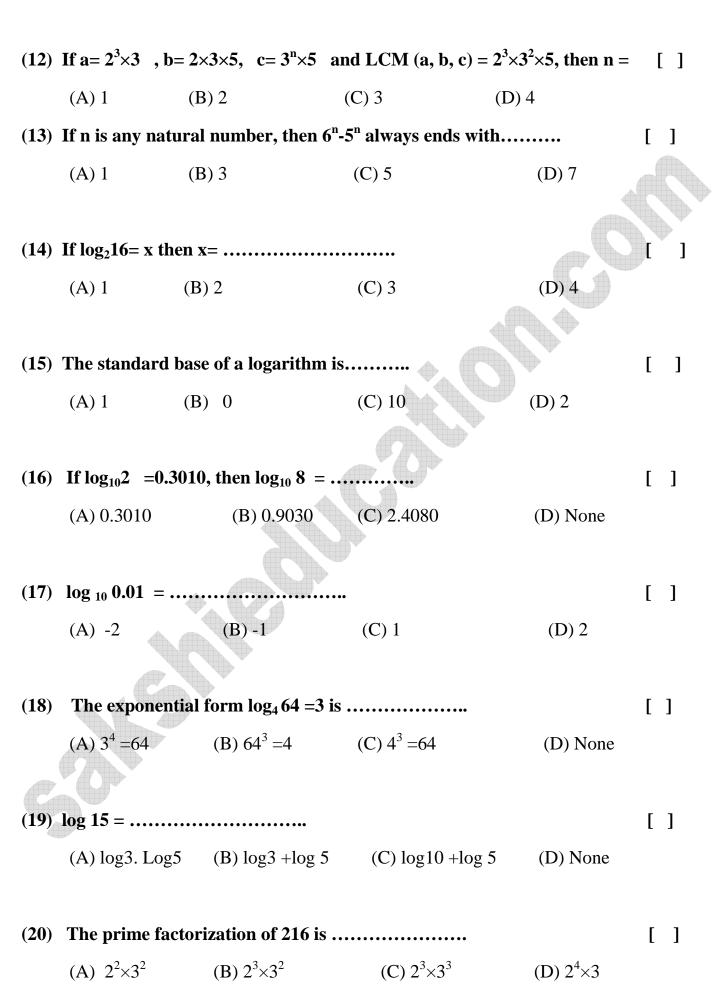
(C) X=21, Y=25

- (D) X=10, Y=40
- - (A) 50
- (B) 8

- (C) $\frac{1}{5}$
- (D) 1
- - (A) An irrational number

(B) a rational number

	(C) a prime number (D) a composite number		osite number		
(6).	which of the following is not an irrational number? []				
	$(\mathbf{A}) \sqrt{2} \qquad \qquad (\mathbf{B})\sqrt{3}$	(C) $\sqrt{4}$	4) √5		
(7).	The reciprocal of two irrational numbers is []				
	(A) Always rational number			Acquire Control of the Control of th	
	(B) Always an irrational number				
	(C) Sometime a rational number, Sometime an irrational				
	(D) Not a real number				
(8).	. The decimal $\frac{17}{8}$ expansion of is	s			
	(A) 2.125 (B) 2.25	(C) 2.375	(D) 2.0125	[]	
(9)	2.547 ⁻ is			[]	
	(A) An integer (B) An irrational	l (C) A rational	(D) A prime nu	ımber	
(10)	Decimal expansion of number $\frac{1}{2}$			[]	
G	(A) A terminating decimal	(B) non –terr	ninating but repeating	ng	
	(C) Non – terminating, non –repeat	ting (D) Termina	ting after two place	s of decin	ıal
(11)) The decimal expansion of $\frac{189}{125}$ w	ill terminate after	·	[]	
	(A) 1 place of decimal (B) 2	places of decimal			
	(C) 3 places of decimal (D) 4	l places of decimal			



Key:

6. C **7.** B **9.** C 10. B **1.**D 2. D 3. B **4.** B 5. A 8. A 11. C 12. B 13. A 14. D 15. C 16. B 17. B 18. C 19. B 20. C

Fill In the Blanks

(1) HCF of 4 and 19 is1......

(2) LCM of 10 and 3 is30......

- (3) If the HCF of two numbers is '1', then the two numbers are called **Co-Prime**.
- (4) If the positive numbers a and b are written as $a = x^5y^2$, $b = x^3y^3$, where x and y are prime numbers then the HCF (a, b),LCM(a, b) = x^3y^2 , x^5y^3
- (5) The product of two irrational number is **Sometimes rational**, **Sometimes**irrational.
- (6) $43.\overline{1234}$ is **a rational** number.
- (7) $\text{Log } a^p$. $b^q = \text{plog } a + q \text{ logb}$
- (8) If $5^3 = 125$, then the logarithm form $log_5 = 125 = 3$
- (9) Log $_{7}$ 343 = 3
- (10) $Log_{2015} 2015 = 1$